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Analysis of Errors in the Semiotic Representation of Fractions among Elementary School Students

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Abstract

This study aims to analyze elementary students' semiotic representation errors in fractions through tasks requiring the conversion of area diagrams into symbolic fraction forms. Using a qualitative case study design, two students were selected as participants. Data were collected through fraction worksheets consisting of five sets of circular area diagrams with shaded parts, supplemented by written documentation of students' responses. Data analysis was conducted through four stages: (1) identifying students' answers, (2) coding error types based on semiotic representation theory, (3) examining unit coherence and fraction structure, and (4) determining error patterns for each subject. The findings show that both students experienced difficulties in identifying the whole and in understanding the relationship between shaded parts and the total number of equal parts. The first subject (S1) correctly identified the numerator but assigned the denominator as the total number of small segments across the entire row of diagrams, resulting in fractions less than one although the context represented improper fractions. The second subject (S2) displayed more complex errors by using different units for the numerator and denominator, leading to fractions that lacked valid mathematical meaning. These errors stem from misconceptions of units, confusion in the roles of numerator and denominator, and challenges in converting visual representations into symbolic forms. The results highlight the need for instructional approaches that strengthen students' understanding of units, fraction structures, and representational flexibility.
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Keywords: Semiotic Representation; Fractions; Student Errors; Numerator–Denominator; Representation Analysis.

1. Introduction

Understanding fractions is one of the key competencies that determines students' success in advanced mathematics, including algebra, proportional reasoning, and ratios. Recent studies emphasize that the ability to understand fractions plays a central role in the development of overall mathematical competence (Bailey et al., 2012; R. S. Siegler et al., 2011). This is reinforced by Park and Matthews (2025), who argue that knowledge of fractions is widely recognized as a fundamental foundation for developing mathematical competence. However, numerous studies indicate that fractions are among the most difficult concepts for students to master, even up to the secondary school level. Cauté et al. (2026) emphasize that "Understanding fractions is a major hurdle for many students." They reported very high error rates, reaching 80% among sixth-grade students and remaining high even as students advanced to higher levels. This evidence aligns with earlier research showing that many students fail to perform basic fraction tasks, such as comparing fractions, placing fractions on a number line, and writing fraction representations correctly (Hannula, 2003; R. Siegler & Pyke, 2019).

One major source of students' difficulties is failure to understand fraction representations. Fractions consist of multiple subconstructs, such as part–whole, operator, quotient, ratio, and measurement (Kieren, 1980; Lamon, 2020). Students' inability to connect these subconstructs often manifests as errors in both visual and symbolic representations. This is confirmed by Herreros-Torres et al. (2026), who show that students experience substantial difficulty when required to shift across fraction representations, ranging from graphical forms and symbolic notation to word problems. Their findings indicate that representational flexibility is a key factor in successful fraction understanding, and that problem-based learning helps learners connect symbolic and graphical representations.

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According to Duval's Theory of Semiotic Representation (Duval, 1993, 2006), successful mathematics learning is strongly determined by students' ability to engage in two core cognitive activities: **treatment** (processing within the same representation, for example, counting parts in a diagram) and **conversion** (transforming one representation into another, such as from a diagram to the symbol a/b). In the context of fractions, conversion is particularly crucial—namely, the ability to transform a visual diagram into the symbolic fraction a/b. Failure in this process often produces systematic errors, such as identifying an incorrect denominator, writing a single whole number, or misunderstanding the part–whole relationship.

Errors in fraction representation among elementary school students mainly occur at the conversion stage, when students fail to translate visual representations (e.g., shaded circle diagrams) into symbolic fractions. This is consistent with the findings of Cauté et al. (2026), who reported that most fraction errors arise from "bugs in the execution of conversion strategies."

Cauté et al. (2026) also found that students frequently make inversion errors, confuse the numerator and denominator, and miscount units when partitions are large in number. They noted that many students struggle to distinguish the roles of the numerator and denominator. Although their study focused on number line tasks, the error models they identified—such as miscounting, inversion, and misunderstanding of units—are highly relevant to picture-based representation tasks, including the circle diagrams used in the present study.

The influence of mathematical language on fraction representation is also substantial. Beyond visual representation, recent research indicates that mathematical language skills play a major role in successful fraction representation. Park and Matthews (2025) demonstrate that mathematics vocabulary is the most significant predictor of students' fraction abilities. They state that "Mathematics vocabulary was the only predictor… that significantly predicted symbolic fraction abilities." This implies that difficulties in writing fractions such as 3/8, 5/10, or 2/6 may stem from limited understanding of the terms numerator and denominator, inadequate grasp of the meaning of "per" or "part of a whole," and weak verbal ability to express fractions. Thus, errors in fraction representation are not only visual but also linguistic in nature.

Although many studies have analyzed fraction errors in number line contexts (Cauté et al., 2026), examined verbal skills related to fractions (Park & Matthews, 2025), explored fraction-as-operator representations (Herreros-Torres et al., 2026), and identified longstanding fraction misconceptions such as whole number bias (Ni & Zhou, 2005), errors in unit partitioning (Steffe & Olive, 2010), and part–whole misconceptions (Kieren, 1980), relatively few studies have directly analyzed elementary students' semiotic representation errors using picture- or diagram-based tasks that integrate visual, symbolic, and linguistic errors in a unified manner. Yet examining students' learning processes is essential (Sutamrin & Khadijah, 2021). This study is therefore intended to address this gap. Accordingly, it aims to identify types of semiotic representation errors in fractions among elementary school students and to analyze error patterns that emerge when students convert visual images/diagrams into symbolic fractions.

2. Literature Review

Jordan et al.'s longitudinal study (Jordan et al., 2017) shows that fraction ability in the early grades is a strong predictor of mathematics achievement at the secondary level. International research indicates that fractions are among the most difficult concepts for students to understand (R. S. Siegler & Lortie-Forgues, 2017; Stafylidou & Vosniadou, 2004). In a large experimental study, Cauté et al. (2025) emphasize that understanding fractions constitutes a major obstacle for many students, and that more than half of students still exhibit systematic errors on number line tasks. These difficulties are closely related to understanding the structure of fractions as part–whole relations, ratios, and operators (Kieren, 1980; Lamon, 2020), as well as how students interpret the relationships among the numerator, denominator, and the total unit.

Duval's theory of semiotic representation (Duval, 1993, 2006) asserts that mathematical objects can only be understood through representations. Two main cognitive processes underlie mathematics learning:

a. Treatment, which involves manipulating information within a single type of representation (for example, counting parts in a diagram).

b. Conversion, which involves transforming representations from one register to another, for instance converting a diagram into a symbolic representation.

Duval (2006) argues that conversion is the most common source of errors because it requires students to coordinate meanings across registers. The findings of Cauté et al. (2026) reinforce this claim by showing that most fraction errors on number line tasks arise from strategic conversion bugs, rather than mere miscalculations. Recent studies by Lima (2023) and Viseu (2021) further indicate that semiotic representation is highly relevant to fraction instruction in the early grades, as students at this age are still in the initial stages of coordinating verbal, visual, and symbolic representations.

Natural Number Bias (NNB) refers to the tendency to apply whole-number rules when working with fractions (Ni & Zhou, 2005). This bias leads students to assume that a fraction with a larger numerator must be larger, to treat fractions merely as "the number of parts" without considering the whole, and to view the numerator and denominator as two separate numbers rather than a single relational ratio (Reinhold et al., 2020). Stafylidou and Vosniadou (2004) show that conceptual change from whole numbers to rational numbers requires students to reconstruct their mental representations. Obersteiner (2020) emphasizes that NNB appears across educational levels and contributes to errors in both fraction comparison and the placement of fractions on number lines.

3. Methods

This study employed a qualitative descriptive approach to analyze patterns of errors in the semiotic representation of fractions based on elementary students' written work. This approach was selected because it enables an in-depth exploration of recurring error patterns and allows them to be interpreted through Duval's theory of semiotic representation as well as recent empirical findings on students' understanding of fractions. The participants were two lower-grade elementary school students (Grades 2–3) who completed fraction tasks based on circle diagrams. The students were purposively selected because they displayed varied representation error patterns that reflect common characteristics of early fraction misconceptions.

Data were collected using a worksheet containing circle diagrams, each divided into several partitions with a number of shaded parts. Students were asked to write the corresponding fraction represented by each diagram. This instrument is aligned with semiotic representation theory because it requires students to convert a visual representation (the diagram) into a symbolic representation (a/b). Conceptually, the task is also closely related to the part–whole subconstruct of fractions (Kieren, 1980).

Data analysis was conducted in five main stages using a qualitative error analysis approach grounded in Duval's theory of semiotic representation (Duval, 1993, 2006). These stages included:

a. Transcribing and documenting students' work from photographs.
b. Item-by-item error identification. Each response was coded based on observable surface errors.
c. Error categorization using Duval's representational framework (semiotic coding), including:
   - Treatment errors (errors in processing information within a single representation),
   - Conversion errors (errors in transforming one representation into another),
   - Unit Identification Errors (errors in understanding units, as discussed by Herreros-Torres et al., 2026), and
   - Numerator–Denominator Confusion (a fundamental error type identified by Cauté et al., 2026).
d. Cognitive interpretation. At this stage, the underlying causes of each error category were analyzed using relevant theory and empirical evidence.
e. Pattern building at both individual and group levels.

Overall, this approach integrates the analysis of visual, symbolic, and linguistic representations in a coherent and unified manner.

4. Hasil dan Pembahasan

This study involved two elementary school student participants (S1 and S2). Both completed five fraction items based on circle diagrams. The response patterns indicate that both students experienced difficulties with semiotic representation; however, the nature of their errors differed. The analysis of students' work revealed four main categories of errors consistent with the theory of semiotic representation and supported by related empirical findings.

4.1. Analysis of Strategies and Errors of Subject 1 (S1)

S1's Work Results:

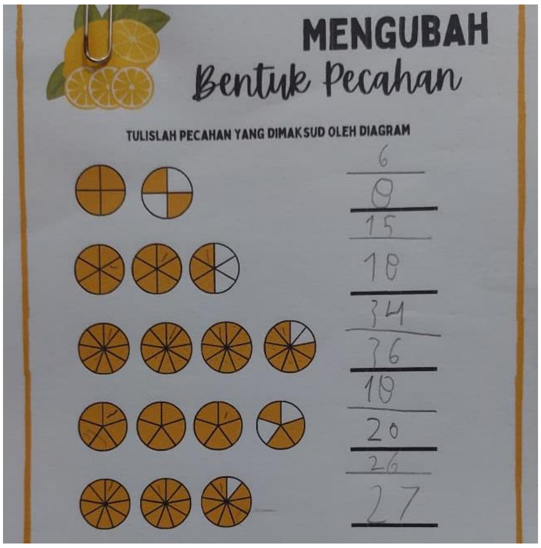

Figure 1. S1's Errors in Converting Fractions

Based on Figure 1, S1 appears to maintain the numerator but select an incorrect denominator. Interpreting the responses, the correct solution for the first row is 6/4, meaning that there are six "quarters" shaded, with each circle partitioned into four parts. S1 wrote 6/8, seemingly assuming that 8 represents the total number of pieces in one row (2 circles × 4 parts). The same pattern appears in the other rows:

Row 2: correct 15/6, S1 wrote 15/18

Row 3: correct 34/9, S1 wrote 34/36

Row 4: correct 18/5, S1 wrote 18/20

Row 5: correct 26/9, S1 wrote 26/27

This indicates that S1 understands the numerator as the number of shaded parts (in small units). However, S1 interprets the denominator as the total number of small pieces across the entire row of images. This suggests that S1 grasps the idea of "part of a whole," but conceptualizes the whole as the combination of all circles in a row, rather than treating each circle as a consistent whole.

Types of Errors Identified in S1

Based on the data, several error types can be identified:

a. Unitizing Error / Whole Misidentification. S1 treated the entire row of images as a single "whole." In the task context, however, each circle should be regarded as a unit that must be preserved. This aligns with a failure in unitizing, where the student is unable to maintain the appropriate unit.

b. Part–Whole Representation Error at the Global Level. In ratio form, S1's fractions still reflect a "part/total" relationship, but the total is incorrect. S1 consistently wrote an a/b form that implicitly means "out of all pieces shown here."

c. Magnitude Error. S1's fractions are consistently < 1, whereas the correct answers are > 1. All correct answers are improper fractions. S1's responses (6/8, 15/18, and so forth) are all less than one. This suggests not only unit misidentification but also difficulty estimating the size of the whole, which may relate to natural number bias and challenges in understanding improper fractions.

d. Conversion Error (Visual to Symbolic) in Duval's Framework. S1 appears able to perform treatment within a single representation (counting shaded parts) but fails in conversion from the visual structure of "multiple circles" to a symbolic fraction that preserves the correct unit (per circle). Thus, the error is not an inability to write a fraction but a mis-modeling of the "whole" during the conversion process.

Overall, S1's profile suggests a developing understanding of fractions as part/total, but with an incorrect total because the whole is interpreted as an entire row rather than a single circle. In other words, S1 can operate with fractions, but the unit of reference is incorrect.

4.2. Analysis of Strategies and Errors of Subject 2 (S2)

S2's Work Results:

Figure 2. S2's Errors in Converting Fractions

Based on Figure 2, S2's answers are **2/6, 3/18, 2/36, 2/20, 1/27**. The pattern indicates that:

a. In rows 2–5, S2's denominators match those of S1 (18, 36, 20, 27).

b. S2's numerators (3, 2, 2, 1) do not appear to represent the number of shaded small parts; instead, they seem to reflect something like the number of circles without shading or the number of "small shading groups."

S2's Use of Multiple, Inconsistent Units. Compared with S1—whose numerator and denominator both refer to "small pieces"—S2 appears to use different units: the numerator seems to count "larger objects" (circles or particular areas), while the denominator is derived from counting "small pieces." This produces unit incoherence, where the numerator and denominator do not operate at the same representational level. Such mismatch constitutes a serious representational error because it undermines the intended ratio meaning of the fraction.

Types of Errors Identified in S2

(a) Unit Mismatch / Incoherent Units. Two levels of units (circles and small partitions) are combined within a single fraction. This is not merely a counting error, but a misselection of the referents for the numerator and denominator.

(b) Numerator–Denominator Role Confusion. Although S2 writes the a/b format, the student does not accurately understand that the numerator represents "how many parts are taken," and the denominator represents "how many equal parts constitute one whole." This aligns with Cauté et al.'s finding that "all grades confused the roles of numerator and denominator."

(c) Two-Level Conversion Error. Misreading the diagram leads to an incorrect determination of what should be counted, followed by a failure to link the count to the a/b structure.

S2's profile suggests that the student understands that the answer should be expressed as a fraction, yet computes the numerator and denominator using different units, resulting in a mathematically incoherent fraction. This error is more conceptual than procedural.

Both students misidentified the whole. S1 treated the whole as all partitions across a row of diagrams. S2 selected the whole in the same way as S1, but calculated the numerator using a different unit. Both therefore demonstrate conversion errors within Duval's categories. Both also failed to recognize that a fraction can be greater than 1. S1's error profile is more procedural with an incorrect unit, whereas S2's is more conceptual but unit-incoherent.

The findings show that both subjects, S1 and S2, committed semiotic representation errors when converting improper fraction diagrams into symbolic mathematical forms. Although both produced answers in the a/b format, there were substantial differences in how they interpreted the numerator (shaded parts) and the denominator (the number of equal parts forming one whole). These errors reflect misconception patterns consistent with the theory of semiotic representation (Duval, 1993, 2006) and international findings on fraction difficulties (Cauté et al., 2026; Herreros-Torres et al., 2026; Park & Matthews, 2025).

From Table 1, S1's error pattern involves accurately identifying the numerator while misidentifying the whole. S1 consistently wrote the numerator correctly according to the total number of shaded small parts across the row. For example, in the first row, S1 wrote 6/8, whereas the correct answer should be 6/4. In the second row, S1 wrote 15/18 (correct: 15/6), and this pattern persisted through the fifth row. The fundamental difference lies in the denominator: S1 did not treat one circle as "one whole," but instead regarded the entire set of circles in a row as the whole. This phenomenon constitutes a form of unitizing error, as discussed in the literature (Obersteiner, 2020; Steffe & Olive, 2010). When a student does not maintain the same unit reference for the numerator and denominator, the fraction no longer represents a valid part–whole ratio.

Table 1. Comparison of S1 and S2

Aspect	S1	S2
Numerator	Correct	Incorrect
Denominator	Incorrect (global unit)	Incorrect (global unit)
Unit Consistency	Incorrect	Highly inconsistent
Fraction Value	Always < 1 (although correct answers > 1)	Always < 1
Main Error Type	Misidentification of the whole	Unit mismatch (∴ fraction lacks meaningful ratio)
Reasoning Stage	"Part/total with an incorrect total"	"A mixture of two unit levels"

S1's case aligns with Herreros-Torres et al. (2026), who report that elementary students often struggle to preserve a single unit when shifting from pictorial to symbolic representations. They note that "learners struggle when the quantitative structure must be represented in continuous-area models," indicating that partition complexity in diagrams can disrupt the identification of the whole. S1 exhibits two major error characteristics:

(a) Magnitude Error (Fraction Size). All correct answers are greater than 1 (improper fractions), such as 6/4 = 1 2/4, 15/6 = 2 3/6, and 34/9 = 3 7/9. However, S1 wrote fractions that were always < 1 (e.g., 6/8, 15/18, 34/36). This indicates that S1 could not recognize that the number of shaded parts can exceed one whole. This error is consistent with Stafylidou and Vosniadou (2004), who show that students often view fractions exclusively as values between 0 and 1 and therefore struggle with improper fractions.

(b) Representation Conversion Error (Visual to Symbolic). Duval (2006) argues that the most frequent mathematical errors occur at the conversion stage, when students transform diagrams into symbols. In S1's case, treatment (counting shaded parts) was successful, but conversion (determining the appropriate whole) failed. Cauté et al.

(2026) similarly emphasize that most fraction errors are caused by “bugs in conversion strategies,” which accurately captures S1’s pattern.

S2’s error pattern involves numerators and denominators derived from different units. Unlike S1, S2 did not consistently preserve the numerator as the number of shaded small parts. For instance, S2’s answers across the rows were 2/6, 3/18, 2/36, 2/20, 1/27. S2’s denominators mirror S1’s denominators (except in row 1), but the numerators are very small, suggesting that S2 may have been counting “shaded circles” or large visual blocks rather than small partitions. This constitutes a unit mismatch, in which the numerator and denominator are computed from different units. Within Kieren’s framework (Kieren, 1980), this reflects a fundamental part–whole misconception: parts and wholes must be defined within the same unit system for a fraction to be meaningful. This observation is consistent with international literature. Winger et al. (2022) found that even pre-service mathematics teachers often fail to maintain unit coherence in fraction representations, especially when the numerator and denominator are interpreted as two separate entities.

S2’s errors appear to involve two major types:

(a) Numerator–Denominator Role Confusion. The numerator should represent the number of small parts taken, yet S2’s numerators appear to represent the number of circles or larger visual groups. This aligns with Cauté et al.’s (2026) observation that students across grade levels “confused the roles of numerator and denominator.”

(b) Fraction Structure Error. Because the units differ, S2’s fractions do not represent a valid mathematical ratio. This type of error commonly arises when students have not yet internalized that a/b means that b equal parts constitute one whole (Lamon, 2020).

Both subjects therefore demonstrate failed visual-to-symbolic conversion—central to Duval’s theory. The errors manifest as incorrect unit selection, flawed fraction structuring, and misinterpretation of diagrams. Both students also rejected the possibility of fractions greater than one (> 1), implicitly assuming that fractions must be “less than one.” This is a strong indicator of natural number bias, namely the transfer of whole-number reasoning to fractions (Ni & Zhou, 2005; Obersteiner, 2020). S2’s structural errors indicate that the student recognizes the a/b format but does not understand the functional roles of numerator and denominator. Park and Matthews (2025) show that limited mathematical vocabulary can contribute to errors in symbolic fractions.

Both students displayed very low representational flexibility when moving from diagrams to symbols and from multiple circles to a consistent unit, consistent with the findings of Herreros-Torres et al. (2026). S1 and S2 failed to establish the “unit whole,” which Steffe and Olive (2010) identify as a core source of fraction misconceptions. Based on the error patterns observed in S1 and S2, several instructional implications follow:

(a) Teachers should explicitly teach what constitutes the “whole,” not only which parts are shaded.

(b) Representations should be explicitly connected rather than assuming students will infer the links—showing a diagram, then writing the fraction, and modeling the diagram from the fraction.

(c) Instruction on improper fractions should be introduced earlier so that students do not remain fixed on the belief that fractions are always < 1.

(d) Practice in mathematical vocabulary is essential (e.g., “shaded parts,” “whole,” “equal parts,” “improper fraction”).

5. Conclusion

Based on the analysis of the two research subjects, this study provides an in-depth depiction of how two elementary school students represent fractions through conversion tasks from images to symbolic forms. The findings indicate that both students still face fundamental difficulties in understanding the relationship between shaded parts and the whole in circle-diagram models. Although both were able to write fractions in the a/b format, their interpretations of the numerator and denominator did not align with the correct concept of fractions.

The first subject (S1) was able to count the shaded parts accurately. However, S1 conceptualized the whole as the combined total of all small partitions across a single row of images, rather than as one complete circle. Consequently, all fractions written by S1 had denominators larger than they should have been. This error resulted in fractions that were consistently less than one, even though the shaded parts exceeded one full circle and should have yielded improper fractions. This suggests that S1 has not yet been able to maintain a consistent notion of the “whole” when dealing with multiple diagrams simultaneously. In contrast to S1, the second subject (S2) displayed a more complex error pattern. S2 not only misinterpreted the whole but also constructed the numerator and denominator from different units. The numerator tended to reflect the number of circles or larger units, while the denominator reflected the total number of small partitions across the entire row of images. As a result, the fractions produced lacked valid mathematical meaning because the numerator and denominator did not refer to the same unit level. This error indicates that S2 has not yet developed an adequate understanding of the basic structure of fractions.

Both subjects also appeared not to understand that fractions can be greater than one. They consistently wrote fractions less than one, even when the shaded portions clearly exceeded the number of parts within a single circle. This pattern suggests a tendency to view fractions only as "parts of one," rather than as rational numbers capable of representing quantities greater than one. This finding is consistent with studies showing that students often transfer whole-number reasoning when working with fractions.

Overall, this study confirms that students' errors in fraction representation are not merely caused by calculation mistakes, but primarily by difficulties in conceptualizing the whole, understanding the functions of the numerator and denominator, and linking visual representations to symbolic forms. These errors suggest that the students' understanding of fractions remains at an early developmental stage, and that they require more targeted instruction to help them identify units, maintain consistency between parts and wholes, and grasp the meaning of improper fractions. The findings carry an important message for teachers: fraction understanding cannot be developed through procedural practice alone. Teachers need to explicitly teach students how to read visual representations, how to determine the whole accurately, and how to convert these representations into mathematical symbols. With appropriate instructional support, students can develop a stronger and more flexible understanding of fractions.

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