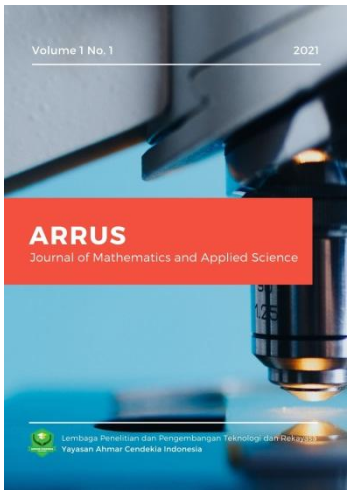
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RESEARCH ARTICLE

From Volatility Shifts to Option Values: A Heston-Dividend Approach to the Indonesian Market*

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Abstract: This study examines how differences in observed volatility conditions are reflected in model-based European option values for Indonesian equities using a Heston-Dividend framework. Daily closing prices of BBCA and TLKM from July 1, 2021, to July 1, 2026, are used to construct daily log returns and annualized 21-day rolling volatility. The observations are divided at January 1, 2026, into an operationally defined baseline period (July 2021–December 2025, n = 1,064) and high-volatility period (January–July 2026, n = 113). Welch's t-test indicates a significant difference in mean volatility between periods ($p < 0.001$) for both assets. Parameter estimation reveals divergent results across periods: for BBCA, reliable mean-reversion parameter estimates could not be obtained using the applied estimation procedure during the high-volatility period, while for TLKM, the estimated mean-reversion speed κ for TLKM decrease from 5.11 in the baseline period to 0.48 in the high-volatility period; the corresponding high-volatility specification does not satisfy the Feller condition. Under the TLKM High-volatility specification, call and put option values are approximately 220–236% higher than their baseline counterparts across the maturities considered. Monte Carlo estimates were compared with the closed-form Heston model, and put–call parity was evaluated using a standardized Monte Carlo parity error at the 5% significance level. The results illustrate how historical volatility estimates can affect scenario-based option valuation when sufficiently liquid option-price data are unavailable.

Keywords: Heston-Dividend Model, Volatility Conditions, European Option Pricing, Monte Carlo Simulation, Indonesian Equity Market.

1. Introduction

Option pricing under stochastic volatility continues to attract attention in financial economics, particularly for markets where derivative instruments remain underdeveloped and calibration data are limited. The Heston (1993) model extends constant-volatility frameworks by specifying variance as a mean-reverting stochastic process, thereby accommodating volatility clustering and the correlation between asset returns and variance changes. For dividend-paying equities, continuous dividend yields are incorporated into the risk-neutral dynamics to reflect the impact of dividend payments on option values (Frontczak, 2011).

The Indonesian equity market offers a useful setting for examining stochastic volatility dynamics. Over the period July 2021 to July 2026, 21-day rolling volatility for BBCA reached a maximum of 70.56%, while TLKM attained 89.56%. These observations suggest the presence of periods with notably different volatility characteristics. Additionally, Dewi and

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Ramli (2018) examine Indonesian stock-option trading using data from 2007–2008 and describe the market as no longer functioning. Accordingly, sufficiently liquid current option quotes are not available for conventional market-price calibration in the present study.

When the full sample is used for parameter estimation, the resulting mean-reversion estimates may not adequately capture the different volatility behaviors observed across the sample period. This limitation motivates dividing the sample into periods with relatively lower and higher observed volatility, allowing separate parameter estimation and comparison of the resulting option values. The period division serves as an operational framework for comparative analysis rather than an attempt to identify the onset of a specific market event.

This study has three objectives: (i) to estimate Heston–Dividend parameters separately for periods with different observed volatility levels; (ii) to document where the applied estimation procedure fails to produce a complete parameter set and to assess the Feller condition when the required parameters are available; and (iii) to compute European option pricing and evaluate numerical consistency using Monte Carlo diagnostics.

2. Literature Review

This section examines the Heston stochastic volatility framework, parameter estimation approaches, and numerical methods for option pricing. Particular attention is given to the limitations of estimation based on historical data and the challenges of applying stochastic volatility models in markets where option-price data are limited or unavailable.

2.1. The Heston Framework and Dividend-Paying Equities

Heston (1993) extends the Black–Scholes framework by specifying instantaneous variance as a mean-reverting square-root process. The model captures time-varying volatility and allows for correlation between asset-price and variance innovations. Its characteristic-function representation enables semi-analytical valuation of European options, making it a widely used benchmark for stochastic-volatility modelling (Frontczak, 2011; Teng et al., 2018).

For dividend-paying equities, the risk-neutral asset-price dynamics incorporate a continuous dividend yield in the drift term. Frontczak (2011) presents an alternative analytical representation for European options in the Heston framework and compares its numerical accuracy with the conventional Fourier-inversion approach. Analytical tractability, however, does not remove the need for reliable parameter inputs.

2.2. Parameter Estimation and Volatility Dynamics

Parameter estimation in stochastic volatility models follows two broad approaches: calibration to observed option prices, and statistical estimation from historical return or variance data. Option-price calibration identifies parameters consistent with market valuations, whereas statistical estimation infers the underlying dynamics from historical observations. Aït-Sahalia and Kimmel (2007) investigate maximum likelihood estimation for stochastic volatility models and compare treatments of the unobserved volatility state, illustrating the methodological complexity inherent in statistical estimation.

For discretely observed Cox–Ingersoll–Ross (CIR) processes, Overbeck and Rydén (1997) study conditional least-squares and likelihood-based estimators. The present study instead uses an AR(1) regression on a rolling-variance proxy; the continuous-time mapping is valid only when the estimated slope satisfies $0 < \hat{\beta} < 1$ and the implied long-run variance is admissible.

Empirical research indicates that volatility dynamics need not be uniform across time or market conditions. Christoffersen, Jacobs, and Mimouni (2010) compare the empirical performance of alternative stochastic volatility specifications and find that model rankings can depend on the evaluation criteria and sample characteristics. Bandi and Renò (2012) document time variation in leverage effects, showing that the relationship between return and variance innovations can depend on the prevailing variance level. In addition, Bollerslev et

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al. (2018) examine realized volatility patterns across multiple asset classes using high-frequency data, demonstrating the relevance of volatility information for risk assessment. These studies provide motivation for examining whether volatility-related estimates differ across sample periods, although they do not establish appropriate sample boundaries for the Indonesian equities examined here or directly validate the particular period division employed in this study.

2.3. The Feller Condition and Numerical Implementation

The Heston variance process follows a CIR specification characterized by the Feller condition, $2\kappa\theta \geq \sigma^2$. When this condition holds, the variance process remains strictly positive almost surely under the continuous-time dynamics. Violation of this condition implies that variance can reach zero with positive probability; however, such violation does not by itself render the model or its parameters unidentified for pricing purposes.

Numerical simulation introduces additional considerations because standard discretization can generate negative variance values even when the underlying continuous-time process is nonnegative. Lord et al. (2010) compare alternative treatments for negative simulated variances and introduce the full-truncation scheme, which reduces the positive pricing bias associated with conventional Euler discretization. Andersen (2008) proposes efficient time-discretization and Monte Carlo simulation algorithms for Heston-type stochastic-volatility models, emphasizing computational efficiency and numerical robustness. These studies demonstrate that numerical results depend on the discretization method, particularly for challenging parameter configurations. Consequently, agreement between simulation estimates and analytical prices, along with appropriate convergence diagnostics, is important when assessing the reliability of reported option values.

2.4. Emerging Market Applications and Research Contribution

Applying stochastic volatility models in markets with limited derivative trading presents distinct challenges. Dewi and Ramli (2018) examine Indonesian stock options using data from 2007–2008, when the market was still active, and describe the market as no longer active. In the absence of sufficiently liquid option prices for calibration, historical-data-based estimation provides an alternative for investigating the variance dynamics of underlying equities. However, parameters estimated from historical observations should be interpreted as statistical estimates rather than parameters calibrated to current option-market prices.

Recent Indonesian studies demonstrate the applicability of Heston-based approaches to different valuation problems. Harahap and Artiono (2025) apply the Heston model to USD/IDR foreign exchange options and compare Fourier-based valuation with Monte Carlo simulation, while Hidayanti et al. (2025) use Heston-based Monte Carlo simulation to price fixed-lookback options using Indonesian equity data. Unlike these studies, the present research examines how Heston-Dividend parameter estimates and European option values differ across periods with different observed volatility levels for BBCA and TLKM. It also documents limitations of the applied estimation procedure, including cases of inadequate parameter identification and violations of the Feller condition. The periods are used as an operational framework for comparison rather than as a formal statistical identification of regime changes.

3. Research Method and Materials

Daily closing prices of BBCA and TLKM from July 1, 2021, to July 1, 2026, are used to construct return and 21-day rolling volatility measures. Daily log returns are calculated as

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right), \tag{1}$$

where P_t denotes the closing price at time t. Annualized historical volatility is then estimated using a 21-trading-day rolling standard deviation, with 252 trading days assumed per year.

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This variance proxy is calculated over the entire observation period before the sample is divided into subsamples using the following formula

$$\hat{\sigma}_{21,t} = \sqrt{252}\, s_{21,t}, \quad \hat{v}_t = \hat{\sigma}^2_{21,t}, \tag{2}$$

where $s_{21,t}$ is the sample standard deviation of the last 21 log returns, and $\hat{v}_t$ is a proxy for the annual variance. The sample is divided at January 1, 2026, into baseline and high-volatility periods. This boundary is used as an operational framework for comparing volatility characteristics, parameter estimates, and option values; it is not intended to identify a statistically determined change point. Historical dividend information is incorporated through the continuous dividend yield (q), while the Bank Indonesia rate is used as a proxy for the risk-free rate (r_f).

Based on this partition, the Heston stochastic-volatility framework with a continuous dividend yield is applied to estimate model parameters and value European options separately for each period. Under the risk-neutral measure $\mathbb{Q}$, the underlying asset price follows the dynamics specified below (Chalimatusadiah et al., 2021; Rouah, 2013)

$$dP_t = (r_f - q)P_t\, dt + \sqrt{v_t}P_t\, dW_t^{(1)}, \tag{3}$$

while the variance process follows the mean-reverting CIR specification (Chalimatusadiah et al., 2021; He & Zhu, 2018)

$$dv_t = \kappa(\theta - v_t)\, dt + \sigma\sqrt{v_t}\, dW_t^{(2)}. \tag{4}$$

Correlation coefficient between the asset-price and variance shocks

$$dW_t^{(1)}\, dW_t^{(2)} = \rho\, dt, \tag{5}$$

where:

κ : mean-reversion speed

θ : long-run variance

σ : volatility of variance

ρ : correlation coefficient, $\rho \in [-1,1]$

v_t : variance, $v_0 > 0$

The model parameters are estimated from the historical variance of the underlying assets instead of calibrating them to observed option prices. This approach is necessary because there is limited trading activity in Indonesian Stock Option Contracts (KOS), so the resulting values are scenario-based model prices rather than market-calibrated risk-neutral quotes. To estimate these parameters, we use the conditional moments of the CIR variance process. We apply an AR(1)-based regression to find the mean-reversion parameters,

$$\hat{v}_t = \alpha + \beta\hat{v}_{t-1} + \varepsilon_t, \tag{6}$$

where $\beta = e^{-\kappa\Delta t}$ and $\alpha = \theta(1-\beta)$. Applying Ordinary Least Squares (OLS) to the AR(1) representation yields estimates $\hat{\alpha}$ and $\hat{\beta}$. The continuous-time parameters are then recovered as

$$\hat{\kappa} = -\frac{\ln\hat{\beta}}{\Delta t}, \qquad \hat{\theta} = \frac{\hat{\alpha}}{1-\hat{\beta}}, \tag{7}$$

provided that $0 < \hat{\beta} < 1$ and $\hat{\alpha} > 0$ with $\Delta t = \frac{1}{252}$. If these conditions are not met, the applied mapping does not produce an admissible (κ, θ) pair for that subsample. Since the

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regression is fitted to a rolling variance proxy rather than directly observed instantaneous variance, these estimates are interpreted as proxy-based estimates of the variance dynamics (Okhrin et al., 2025).

Then, we use the residual structure $\hat{\varepsilon}_t$ obtained from the fitted AR(1) regression in Equation (6) are used to estimate the other variance parameters $\hat{\sigma}$ and ρ as follows

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{t=1}^{n}\frac{\hat{\varepsilon}_t^2}{\hat{v}_{t-1}\Delta t}. \tag{8}$$

$$\hat{\rho} = corr\left(z_t^{(1)}, z_t^{(2)}\right), z_t \sim N(0,1), \tag{9}$$

where the variables $z_t^{(1)}$ and $z_t^{(2)}$ denote the standardized log returns and standardized residuals from the AR(1) regression in Equation (5), respectively. Then, the initial variance v_0 is set equal to the last valid observation of the historical variance proxy within each subsample. This initialization rule allows each pricing scenario to reflect the variance level observed at the end of its corresponding period.

The resulting parameter estimates are then used to represent the volatility conditions considered in the pricing analysis. The Feller condition $2\kappa\theta \geq \sigma^2$, is also considered when assessing the admissibility of the estimated variance dynamics.

The European option pricing is defined under the risk-neutral measure as the discounted expected terminal payoff,

$$\begin{aligned} C_0^{call} &= e^{-r_f T}\mathbb{E}^{\mathbb{Q}}[\max(P_T - K, 0)] \\ C_0^{put} &= e^{-r_f T}\mathbb{E}^{\mathbb{Q}}[\max(K - P_T, 0)] \end{aligned} \tag{10}$$

Following the Heston formulation with continuous dividend adjustment, the closed-form values are expressed as

$$\begin{aligned} C_0^{call} &= P_0 e^{-qT}\Pi_1 - Ke^{-r_f T}\Pi_2 \\ C_0^{put} &= C_0^{call} - P_0 e^{-qT} + Ke^{-r_f T} \end{aligned} \tag{11}$$

where:

$$\Pi_j = \frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} Re\left[\frac{e^{-i\phi \ln K} f_j(\phi)}{i\phi}\right] d\phi,$$

and $f_j(\phi)$ is the characteristic function associated with the Heston option pricing model.

The Monte Carlo valuation follows a Milstein discretization with full truncation to preserve the non-negativity of the variance process (Cao & Lin, 2024; Rouah, 2013). The overall computational workflow, including simulation, payoff evaluation, and comparison with the closed-form benchmark, is summarized in Figure 1.

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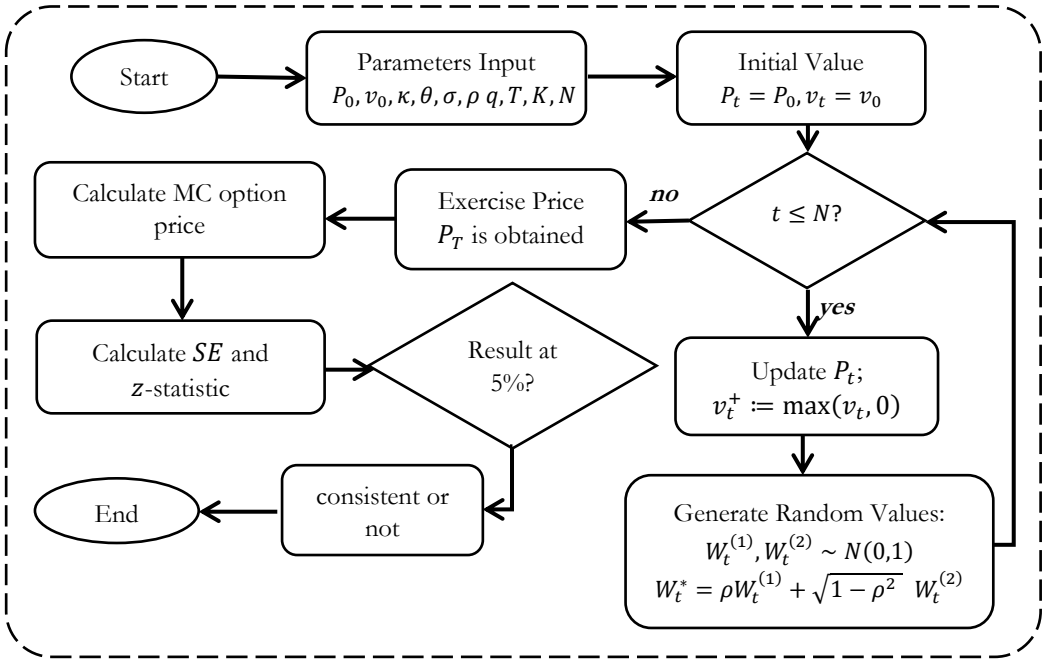


Figure 1. Monte Carlo Simulation Flowchart

Put–call parity is evaluated by comparing the Monte Carlo estimate of the call–put price difference with its theoretical value under the parity relationship. The closed-form and Monte Carlo price differences are defined as

$$D_{CF} = P_0 e^{-qT} - K e^{-r_f T} \text{ and}$$
$$\widehat{D}_{MC} = C_0^{call} - C_0^{put}, \tag{12}$$

respectively. For independently estimated call and put prices, the standard error of their difference is calculated as

$$SE = \sqrt{SE_{call}^2 + SE_{put}^2}\,. \tag{13}$$

The standardized deviation from the theoretical parity value is then computed as

$$z = \frac{\widehat{D}_{MC} - D_{CF}}{SE}\,. \tag{14}$$

At the 5% significance level, a value of $|z| \leq 1.96$ indicates that the null hypothesis of no statistically significant deviation from put–call parity is not rejected. Furthermore, a sensitivity analysis is conducted by varying selected Heston parameters to assess the stability and robustness of the resulting option values under alterations in the variance dynamics.

4. Results and Discussion

This section presents the empirical findings on historical volatility, Heston-Dividend parameter estimates, and European option values for BBCA and TLKM. The results are followed by an assessment of numerical consistency and their implications for model-based option pricing.

4.1. Historical Volatility

Figure 2 presents the 21-day rolling volatility of BBCA and TLKM from July 2021 to July 2026. Both series exhibit fluctuations throughout the observation period, with more pronounced increases toward the end of the sample, particularly for TLKM. The maximum rolling volatility over the full sample reached 70.56% for BBCA and 89.56% for TLKM. The

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vertical dashed line marks January 1, 2026, which serves as the operational boundary between the baseline and high-volatility periods in the subsequent analysis.

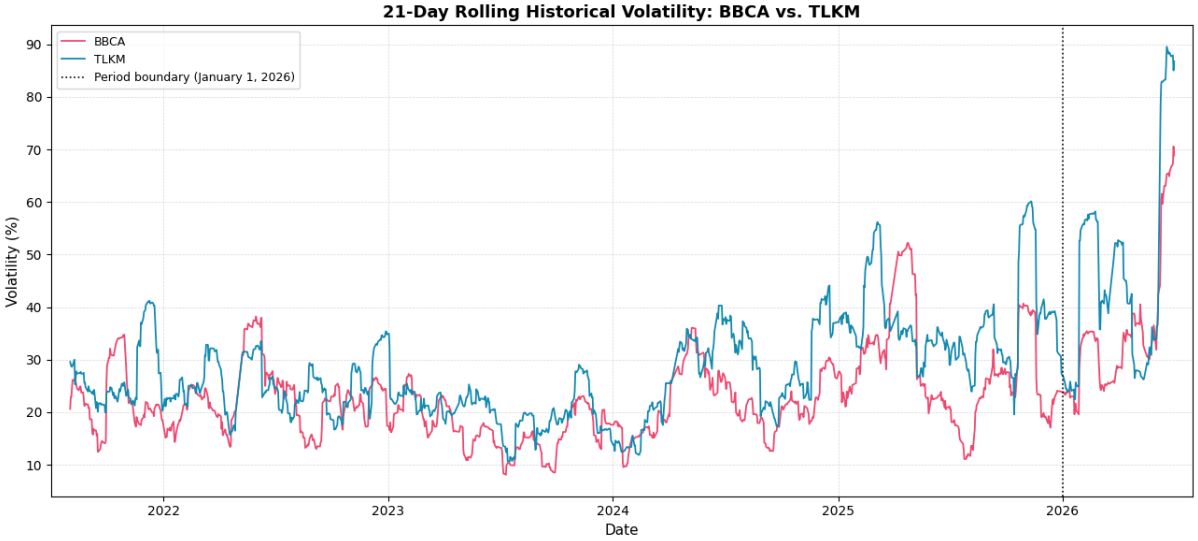


Figure 2. 21-Day Rolling Volatility of BBCA and TLKM, July 2021-July 2026.

To quantify these visual patterns, Table 1 summarizes the distribution of 21-day rolling volatility for each asset and period using the mean, standard deviation, minimum, and maximum.

Table 1. Historical Volatility Characteristics by Asset and Period

Asset and Period	*N*	Mean (%)	Std. Dev. (%)	Minimum (%)	Maximum(%)
BBCA - Baseline	1064	22.19	7.56	8.09	52.22
BBCA - High-Vol	113	35.44	12.89	19.59	70.56
TLKM - Baseline	1064	27.49	8.99	10.61	60.12
TLKM - High-Vol	113	46.71	19.63	22.44	89.56

Both assets exhibit higher mean rolling volatility and greater dispersion during the high-volatility period than during the baseline period. The standard deviation increases from 7.56 to 12.89 percentage points for BBCA and from 8.99 to 19.63 percentage points for TLKM, indicating a more pronounced increase in dispersion for TLKM. The minimum volatility values are also higher in the high-volatility subsamples for both stocks. Taken together, these descriptive statistics indicate that the selected high-volatility period is characterized by both elevated volatility levels and greater variability in the rolling-volatility estimates.

To assess whether the differences in mean rolling volatility between the two periods are statistically significant, Welch's t-test is applied separately to BBCA and TLKM, with the results presented in Table 2.

Table 2. Statistical Comparison of Mean Rolling Volatility

Asset	Baseline Period (%)	High-Vol Period (%)	Change (%)	Welch's t-test	Welch's p-value
BBCA	22.19	35.44	+59.73	-10.7381	< 0.001
TLKM	27.49	46.71	+69.91	-10.2948	< 0.001

Welch's t-test yields p-values below 0.001 for both assets, indicating statistically significant differences in mean rolling volatility between the selected periods under the conventional test assumptions. The larger relative increase, 69.91% for TLKM indicates a more pronounced difference in average volatility between the selected periods. These findings support the comparison of mean volatility across the two periods, although they do not establish January 1, 2026, as an optimal change point or identify the causes of the observed increases. Since consecutive 21-day rolling-volatility estimates are based on overlapping return observations, potential serial dependence should be considered when interpreting the conventional Welch's t-test results.

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4.2. Heston-Dividend Parameter Estimates

Table 3 presents the estimated Heston-Dividend parameters for each asset and period. During the baseline period, both assets yield positive mean-reversion estimates and satisfy the Feller condition. Their estimated mean-reversion speeds are similar, with (κ = 5.1845) for BBCA and (κ = 5.1134) for TLKM. Their long-run variance estimates differ, at (θ = 0.0556) and (θ = 0.0830), respectively.

Table 3. Heston-Dividend Parameter Estimates by Asset and Period

Scenario	κ	θ	σ	ρ	v_0	q	Feller
BBCA – Baseline	5.1845	0.0556	0.4986	0.1776	0.0568	0.0258	satisfied
BBCA – High-vol	-	-	-	-	0.4740	0.0258	not assessable
TLKM – Baseline	5.1134	0.0830	0.6161	0.0765	0.0746	0.0555	satisfied
TLKM – High-vol	0.4792	3.4550	1.8900	-0.5211	0.7529	0.0555	not satisfied

For BBCA during the high-volatility period, the estimated AR(1) slope falls outside the interval $(0 < \hat{\beta} < 1)$ required by the discrete-to-continuous parameter mapping used to estimate κ and θ. Consequently, the applied procedure does not yield a complete parameter specification for this subsample. Although v_0 and q are available, the parameters required to complete the Heston specification are not all obtained. The Feller condition is therefore not assessable for this case. This result reflects a limitation of the applied estimation procedure for the selected subsample, rather than evidence that a valid Heston parameter set cannot exist.

For TLKM, the estimated parameters differ between the baseline and high-volatility periods. The estimated mean-reversion speed decreases from 5.1134 to 0.4792, while the long-run variance increases from 0.0830 to 3.4550 and the volatility of variance rises from 0.6161 to 1.8900. The return-variance correlation also changes from positive (ρ = 0.0765) to negative (ρ = -0.5211). These differences indicate that the estimated parameter specifications are sensitive to the selected sample period, but they do not establish a formal structural change in the underlying variance process.

The shorter high-volatility subsample provides less information for estimating the variance dynamics and may make the parameter estimates more sensitive to the observations included. In particular, κ and θ depend on the estimated OLS coefficients, whereas σ and ρ rely on residual-based estimates. Accordingly, these estimates should be interpreted with caution, as their uncertainty was not formally quantified in this study.

The Feller condition is satisfied for both baseline specifications but not for TLKM during the high-volatility period, for which $2\kappa\theta < \sigma^2$. This violation means that strict positivity of the variance process is not guaranteed under the continuous-time model, but it does not by itself invalidate the Heston specification or imply that its parameters are unidentified.

Accordingly, option valuation proceeds using the three specifications with complete parameter estimates: BBCA under baseline conditions, TLKM under baseline conditions, and TLKM under high-volatility conditions. The BBCA high-volatility specification is excluded because the applied estimation procedure does not provide a complete parameter set, whereas the TLKM high-volatility specification is retained despite its Feller condition violation. This distinction separates the completeness of the parameter estimates from the boundary behavior indicated by the Feller condition.

4.3. European Option Valuation

Table 4 reports the closed-form European call and put option values obtained under the three parameter specifications retained for the pricing analysis, across maturities of 1, 3, 6, and 12 months. The BBCA high-volatility scenario is excluded because the applied estimation procedure does not yield a complete parameter specification for that subsample.

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Table 4. European Option Prices under the Retained Parameter Specifications

Scenario	1-Month		3-Month		6-Month		12-Month	
	Call	Put	Call	Put	Call	Put	Call	Put
BBCA – Baseline	158.62	143.92	279.67	235.79	403.93	317.07	587.56	417.41
TLKM – Baseline	76.44	76.04	132.14	130.96	186.58	184.26	260.36	255.85
TLKM – High-vol	244.74	244.34	426.64	425.46	608.39	606.07	864.09	859.58
TLKM – High-vol vs Baseline, Δ (%)	**+220.2**	**+221.3**	**+222.9**	**+224.9**	**+226.1**	**+228.9**	**+231.9**	**+236.0**

Across all three specifications, the reported call and put values increase with maturity. The most pronounced difference is observed between the two TLKM scenarios: relative to the baseline specification, the high-volatility specification produces call values that are 220.2–231.9% higher and put values that are 221.3–236.0% higher. The relative differences also widen with maturity, indicating that the resulting valuations vary substantially with the parameter specification used for TLKM.

These values are conditional model-based valuations derived from historical parameter estimates, rather than prices observed in an active options market. Accordingly, the differences should be interpreted as the effects of applying the two estimated parameter specifications within the Heston-Dividend framework, not as direct estimates of executable market prices or realized trading costs.

4.4. Numerical Validation and Practical Implications

The numerical reliability of the option valuations is examined by comparing Monte Carlo (MC) estimates with the corresponding closed-form (CF) prices across simulation path counts and maturities. Put–call parity and parameter sensitivity are also evaluated to provide complementary checks on the numerical consistency and robustness of the reported valuations.

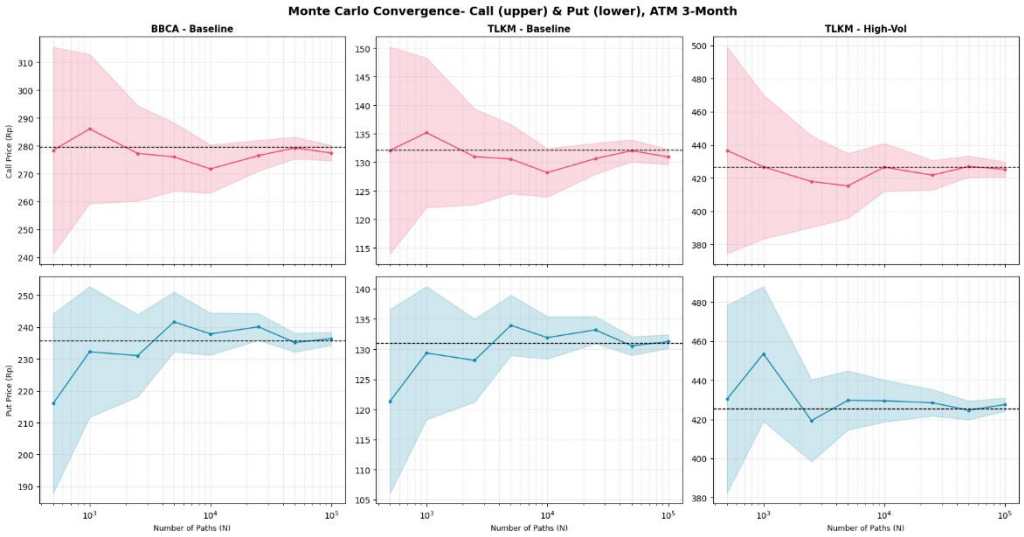


Figure 3. Monte Carlo Price Estimates and Confidence Intervals across Simulation Path Counts

Figure 3 illustrates the behavior of Monte Carlo price estimates as the number of simulation paths increases. The estimates exhibit greater variability at lower path counts, while the associated 95% confidence intervals generally narrow as additional paths are used. At higher path counts, the estimates tend to stabilize around the corresponding closed-form benchmarks. This pattern indicates improved simulation precision, although visual agreement at a finite number of paths alone is insufficient to establish numerical convergence.

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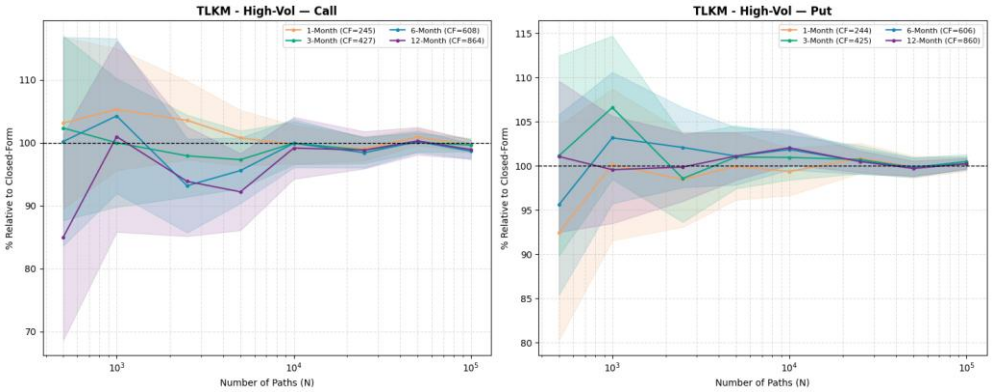


Figure 4. Monte Carlo Prices Relative to Closed-Form Values across Maturities

The comparison across maturities indicates that relative price estimates closer to 100% correspond to closer agreement between Monte Carlo and closed-form prices. The degree of agreement varies across maturities, highlighting differences in the magnitude of numerical discrepancies. Overall, the estimates tend to approach the closed-form benchmarks as the number of simulation paths increases. These findings support the numerical consistency of the pricing implementation under the selected TLKM high-volatility specification, but do not establish the empirical accuracy of the underlying historical parameter estimates (Figure 4).

To complement this comparison, Table 5 reports the Monte Carlo and closed-form prices for three-month at-the-money options under the three retained parameter specifications, together with the corresponding put–call parity diagnostic.

Table 5. Numerical Validation of Option Prices and Put–Call Parity for 3-Month ATM

Scenario	CF call	MC call	CF put	MC put	CF: Call-Put	MC: Call-Put	*z*-statistic	Result at 5%
BBCA – Baseline	279.67	279.87	235.79	235.14	43.88	44.73	0.993	consistent
TLKM – Baseline	132.14	132.38	130.96	131.61	1.18	0.77	-0.942	consistent
TLKM – High-vol	426.64	424.87	425.46	425.76	1.18	-0.89	-1.521	consistent

The Monte Carlo estimates are generally close to their corresponding closed-form benchmarks across the three retained specifications. The reported put–call parity test statistics also remain within the conventional two-sided 5% critical bounds, indicating no statistically significant discrepancy under the stated test procedure. These findings support the numerical consistency of the simulated valuations for the configurations examined. However, agreement between numerical and analytical results does not independently validate the historical parameter estimates or establish that the resulting prices correspond to executable market prices.

To examine local parameter sensitivity, Figure 5 shows how European option values respond to ±20% perturbations in the volatility-of-variance parameter σ and the return–variance correlation parameter ρ under the three retained specifications.

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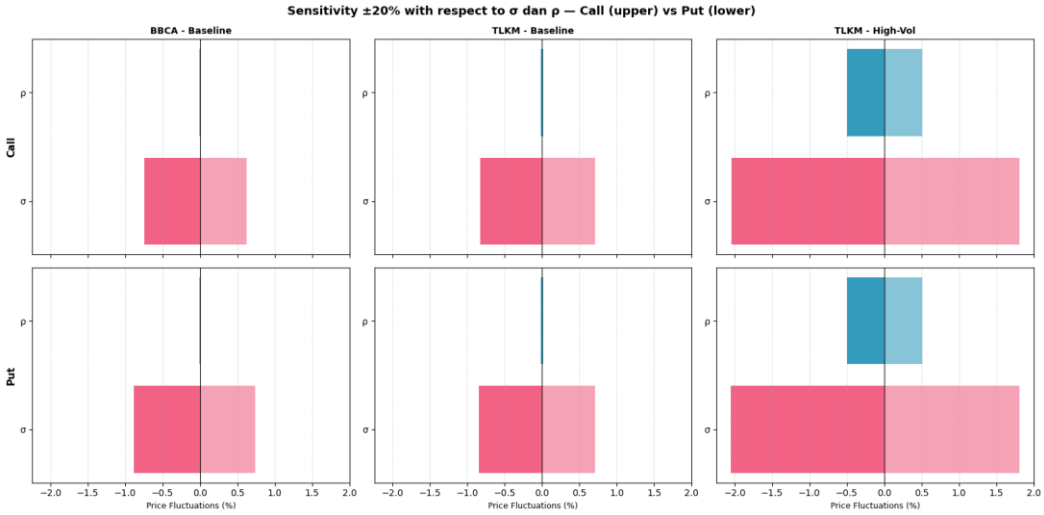


Figure 5. Sensitivity of European Option Values under Baseline and High-volatility Conditions

Within the perturbation range examined, option values exhibit a stronger response to changes in the volatility-of-variance parameter than to changes in the return–variance correlation parameter, particularly under the TLKM high-volatility specification. The baseline specifications display more limited price variation, whereas the high-volatility specification shows greater sensitivity to the tested parameter changes. This pattern suggests that the relative influence of individual parameters depends on the estimated specification. The interpretation is limited to the selected parameter values and ±20% perturbations and should not be generalized beyond the range examined.

Taken together, the numerical comparisons and sensitivity analysis provide complementary evidence on the behavior of the pricing calculations under the retained parameter specifications. For markets with limited option-price data, such analyses can help assess the internal consistency of model-based valuations and identify parameters to which the resulting prices are particularly responsive. Nevertheless, these numerical checks do not substitute for calibration to observed market prices, and the resulting valuations should be interpreted as conditional model-based estimates rather than directly tradable prices.

5. Conclusion

This study compared Heston-Dividend parameter estimates and European option values across baseline and high-volatility periods for BBCA and TLKM. The estimated parameters differ across the selected periods, particularly for TLKM, indicating that model-based valuations are sensitive to the historical variance specification used. Under the TLKM high-volatility specification, the calculated call and put values are approximately 220–236% higher than their baseline counterparts across the maturities examined. For BBCA, the applied estimation procedure did not yield a complete parameter specification for the high-volatility subsample, preventing its inclusion in the subsequent pricing analysis. The relatively short high-volatility subsample warrants cautious interpretation of the estimated parameters and the resulting option values. The numerical comparisons show that Monte Carlo estimates are broadly consistent with the corresponding closed-form prices, while the reported put–call parity diagnostic indicates no statistically significant discrepancy under the test procedure used. These results demonstrate the sensitivity of theoretical option valuations to parameter specification, but do not establish a formal structural change in the underlying variance process or validate the calculated prices against observed market transactions.

The findings may inform traders and investors conducting scenario-based option valuation, as well as financial institutions assessing model risk under alternative volatility assumptions. Given the limited availability of actively traded Indonesian stock options, the resulting values should be interpreted as model-based estimates rather than executable market prices. The findings also highlight the value of accessible and reliable derivatives-price data and

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transparent market information for market participants and regulators in evaluating and validating model-based valuations, although this study does not assess specific regulatory measures. Future research could explore alternative parameter estimation methods, quantify parameter uncertainty, and incorporate observed option prices when reliable market data become available.

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